Specialist Mathematics Unit 1: **Chapter 2**

Ex 2A.

1a) $3!$

$= 3 \times 2 \times 1$

$= 6$

b) $3! + 2!$

$= 3 \times 2 \times 1 + 2 \times 1$

$= 6 + 2$

$= 8$

c) $(3+2)!$

$= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

d) $\frac{11!}{10!}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 11$

e) $\frac{11!}{9!}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 110$

f) $\frac{6!}{4!2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$

$= \frac{30}{2} = 15$

g) ${}^5P_2$

$= \frac{5!}{(5-2)!} = \frac{5!}{3!}$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$

$= 20$

h) ${}^7P_3$

$= \frac{7!}{(7-3)!} = \frac{7!}{4!}$

$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$

$= 210$

i. ${}^8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 56$

Ex 9B.

10. First or second place
$12 \times 11 = 132$

11. code $0 \rightarrow 9$
0 ok to start, no repeats
$10 \times 9 \times 8 \times 7$
$= 5040$

12. 12 horses 6 places
$12 \times 11 \times 10 \times 9 \times 8 \times 7$
$= 665280$

13. 10 ques multichoice of a, b, c, d
$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{10}$
$= 1048576$

14. 15 people 3 places
$15 \times 14 \times 13$
$= 2730$

15. a) 8 goals 3 places
$8 \times 7 \times 6$
$= 336$

b) 8 goals 8 places
$8! = 40320$

16. 10 ques true/false (yes/no)
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 1024$

1. REPEAT (E×2)
$\frac{6!}{2!} = 360$

2. CLASSES (S×3)
$\frac{7!}{3!} = 840$
$=$

3. TROTTER (T×3 R×2)
$\frac{7!}{3!2!} = 420$

4. PERMUTATIONS
(T×2)
$\frac{12!}{2!}$
$= 239500800$

5. MISSISSIPPI
$\frac{11!}{4!4!2!}$ (I×4) (S×4) (P×2)
$= 34650$

Starts with M.
M _ _ _ _ _ _ _ _ _ _
$\frac{1 \times 10!}{4!4!2!}$
$= 3150$

2. 

$3 \times 2 = 6$

3. roll (4) ham (1) lettuce (or not) mustard (or not)

$= 4 \times 1 \times 2 \times 2$
$= 16$

4. ○○○○○○ 6 marbles

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$
minus the one he has arranged
$= 719$ other possible arrangements

5. bus, bike, foot

5 days $\square\ \square\ \square\ \square\ \square$
$3 \times 3 \times 3 \times 3 \times 3$
$= 3^5$
$= 243$

6. FASHION = 7 letter words

a) once only $= 7! = 5040$

b) repeats allowed
$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
$= 7^7 = 823543$

7. FASHION (5 letter word)

a) once only
$\boxed{7} \times \boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3}$
$= 2520$

b) repeats allowed
$\boxed{7}\ \boxed{7}\ \boxed{7}\ \boxed{7}\ \boxed{7}$
$= 7^5$
$= 16807$

8 a) not returned
$15 \times 14 \times 13 = 2730$

b) is returned
$15 \times 15 \times 15 = 3375$

9 11 of each type of ques

a) 5 ques
$12 \times 10 \times 10 \times 6 \times 8$
$= 57600$

b) 8 ques
$12 \times 10 \times 10 \times 6 \times 8 \times 4 \times 9 \times 6$
$= 12441600$

c) all 10 ques
$12 \times 10 \times 10 \times 6 \times 8 \times 4 \times 9 \times 6 \times 5 \times 5$
$= 311040000$

6. WOLLONGONG

L×2 O×3 N×2 G×2

a) $\frac{10!}{2!3!2!2!} = 75600$

b) Start W

W _ _ _ _ _ _ _ _ _

$\frac{1 \times 9!}{2! \times 3! \times 2! \times 2!}$

= 7560

c) not start with W

75600 − 7560

= 68040

7. 1, 2, 3, 4, 5 once only

2 digit + 3 digit

5×4 + 5×4×3

= 20 + 60 = 80

8. A → Z 2 or 3 letter code repeat or

2 + 3

26×26 + 26×26×26

$= 26^2 + 26^3$

= 18252

9. A → Z 2 or 3 letters no repeat

2 + 3

26×25 + 26×25×24

= 16250

10. 1¢2¢3 2 digit & 3 digit

a) repeat allowed

2 digit + 3 digit

3×3 + 3×3×3

= 36

b) no repeats

2 digit + 3 digit

3×2 + 3×2×1

= 12

11. 1 → 5 2 & 3 digit N°

a) repeat allowed

2 digit + 3 digit

5×5 + 5×5×5

= 150

b) no repeats

2 digit + 3 digit

5×4 + 5×4×3

= 80

12. Four character code

1 2 3 4 5 or A B C D E F G

repeat no repeat

5×5×5×5 + 7×6×5×4

= 1465

13. Use MATH or 1 letter missing FUN

5×4×3 + 5×3×2

= 90

14. race 8 or race 9 not both

1st, 2nd 3rd out of 8 — 1st, 2nd out of 12

$8 \times 7 \times 6 + 12 \times 11$

$= 468$

15. long key = 5 Short key = 3

a) repeat allowed

$8 \times 8 \times 8 \times 8 \times 8 + 8 \times 8 \times 8$

$= 33280$

b) no repeats

$8 \times 7 \times 6 \times 5 \times 4 + 8 \times 7 \times 6$

$= 7056$

16. long key = 5 Shot key = 3

9 options.

a) repeat allowed

$9^5 + 9^3 = 59778$

b) no repeats

$9 \times 8 \times 7 \times 6 \times 5 + 9 \times 8 \times 7$

$= 15624$

17. 2 digit code from

$1 \rightarrow 6$ or $6, 7, 8$

no repeats!

$6 \times 5 + 3 \times 2$

$= 36$

18. CREDIT OR COMPANY

4 digit code

no repeats

$6 \times 5 \times 4 \times 3 + 7 \times 6 \times 5 \times 4$

$= 1200$

19. Nifty — 4 colour

2 engine size

AC / no AC

trans / no trans

power / no power

Sedale — 5 colour

3 engine size

1 ac

1 trans

1 power

nifty + sedale

$4 \times 2 \times 2 \times 2 \times 2 + 5 \times 3 \times 1 \times 1 \times 1$

$= 79$

20. 2 different digits

$1 \rightarrow 5$ or $4, 5, 6, 7$

$5 \times 4 + 4 \times 3 = 32$

but have to minus

codes that may overlap

"4" & "5"

ie $2 \times 1 = 2$

$32 - 2 = 30$

21. 3 digit all diff code
1 → 5 or 3, 4, 5, 6

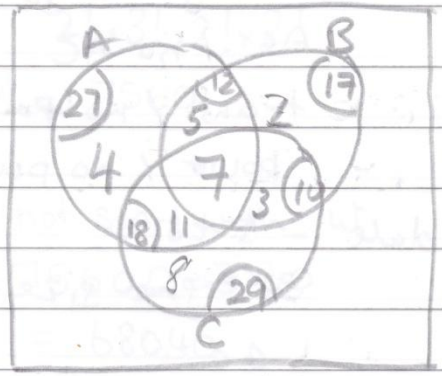
$5 \times 4 \times 3 + 4 \times 3 \times 2 = 84$

- minus the ones that may overlap ie "3" "4" "5"

$\Rightarrow 3 \times 2 \times 1 = 6$

$84 - 6 = 78$

23.



$A \cup B \cup C = 40$

Use formula

$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 27 + 17 + 29 - 12 - 18 - 10 + 7$

$= 40$

22. a) x of 5 $1 \rightarrow 999$ $999 \div 5 = 199$

b) x of 7 $1 \rightarrow 999$ $999 \div 7 = 142$

c) x of 5 or 7 → ie x by 35

$|\frac{x}{5}| + |\frac{x}{7}| - |\frac{x}{5 \cap 7}|$

$999 \div 35 = 38$ $199 + 142 - 28 = 313$

24. $1 \rightarrow 101$

x by 2 $\Rightarrow 100 \div 2 = 50$

x by 3 $\Rightarrow 100 \div 3 = 33$

x by 5 $\Rightarrow 100 \div 5 = 20$

x by 6 $\Rightarrow 100 \div 6 = 16$

x by 15 $\Rightarrow 100 \div 15 = 6$

x by 10 $\Rightarrow 100 \div 10 = 10$

x by 30 $\Rightarrow 100 \div 30 = 3$

ie $50 + 33 + 20 - 16 - 6 - 10 + 3 = 74$

25. $1 \rightarrow 1001 = 1000$ N°

x by 3 = 333

x by 10 = 100

x by 25 = 40

x by 30 = 33

x by 75 = 13

x by 150 = 6 * LCM 3, 10, 25

x by 50 = 20 * LCM 25, 10

$333 + 100 + 40 - 33 - 13 - 20 + 6$

$= 413$

26. $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$

Ex 2C.

1. GREAT 2 letter is G

a) no repeats

$\underset{\text{"g"}}{4} \times 1 \times 3 \times 2 \times 1 = 24$

b) repeats allowed

$\underset{\text{"g"}}{5} \times 1 \times 5 \times 5 \times 5 = 625$

2 a) 1, 2, 3, 4, 5, 6

odds! no repeats

$5 \times 4 \times 3 \times 2 \times 1 \times \underset{1,3 \text{ or } 5}{3}$

$= 360$

b) greater than 600 000

* 1st must be "6"

$\underset{\text{"6"}}{1} \times 4 \times 3 \times 2 \times 1 \times \underset{1,3,\text{or } 5}{3}$

$= 72$

3 a) 6 Files → $6! = 720$

b) D & F together

DF A B C =

$5! \times 2! = 240$

c) "A" "B" "C" exactly in this order

ABC D E F

$4! = 24$

d) ABC but jumbled also

ie $4! \times 3! = 144$

4. X A E I O R $5! = 120$

a) start with consonant

$\underset{\uparrow}{3} \times 4 \times 3 \times 2 \times 1 = 72$

× J, Q, F

b) start with vowel

$\underset{\text{a,e}}{2} \times 4 \times 3 \times 2 \times 1 = 48$

5. 5 names $= 5! = 120$

a) $\underset{\text{Jack}}{1} \times 4 \times 3 \times 2 \times 1 = 24$

b) $\underset{\text{Jill}}{4} \times 1 \times 3 \times 2 \times 1 = 24$

c) $\underset{\text{Jack Jill}}{1} \times 1 \times 3 \times 2 \times 1 = 6$

6. a) 1 → 7 no repeats

$7! = 5040$

b) 1 → 7 even n° no repeats

$6 \times 5 \times 4 \times 3 \times 2 \times 1 \times \underset{2,4,6}{3}$

$= 2160$

c) even larger than 7 000 000

$\underset{\text{"7"}}{1} \; 5 \; 4 \; 3 \; 2 \; 1 \; \underset{\uparrow}{3}$

"2, 4, 6"

$= 360$

7a) Starter x3 Main x4 desert x2

$3 \times 4 \times 2 = 24$

b) has Lasagne

$3 \times 1 \times 2 = 6$

(1 ↑ Lasagne)

c) has Lasagne & ice-cream

$3 \times 1 \times 1 = 3$

Lasagne Ice cream

8. $0 \to 9$ $A \to Z$

3 digit 2 letters

a) no digit repeat letter repeat ok

$10 \times 9 \times 8 \times 26 \times 26$

$= 486720$

b) digit repeat ok letter repeat no!

$10 \times 10 \times 10 \times 26 \times 25$

$= 650\,000$

c) not start 0 & nothing repeat

$9 \times 9 \times 8 \times 26 \times 25$

$= 421200$

d) not start 0, repeat ok, last is vowel

$9 \times 10 \times 10 \times 26 \times 5$

(5: a, e, i, o, u)

$= 117000$

9. coin, die, coin, die

a) $2 \times 6 \times 2 \times 6$

$= 144$

b) die are same

$2 \times 6 \times 2 \times 1$

(1 ↑ same)

$= 24$

c) coins same

$2 \times 6 \times 1 \times 6$

(1 ↑ same)

$= 72$

10. 3 letters & 2 digits

a) no restrictions

$26 \times 26 \times 26 \times 10 \times 10$

$= 1757600$

b) no repeats

$26 \times 25 \times 24 \times 10 \times 9$

$= 1404000$

c) Start not vowel no repeats

$21 \times 25 \times 24 \times 10 \times 9$

(21 ↑ ~~a e i o u~~)

$= 1134000$

d) $26 \times 25 \times 1 \quad 10 \quad 1$

(1 ↑ same; 1 ↑ same)

$= 6500$

e) letters are consecutive reverse not allowed ie P Q not Q P

$24 \times 1 \times 1 \times 10 \times 10$

$= 2400$

cannot start Y or Z — last is only XYZ

f) final digit is one more than previous ie 5, 6
& letters consecutive

$24 \times 1 \times 1 \times 9 \times 1$

↑ end cannot be "9" "8"

$= 216$

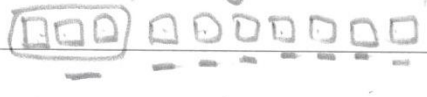
11. 10 books 3 by same person

a) no restrictions

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3628800$

b) author kept together in particular order



8 items $8! = 40300$

c) author together but can shuffle

$8! \times 3! = 241920$

d) author at left end in particular order

$1 \times 1 \times 1 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

Ex 2D.

1. 1 → 5 3 or 4 digits

a) repeat ok

$5 \times 5 \times 5 + 5 \times 5 \times 5 \times 5$

$= 750$

b) no repeats

$5 \times 4 \times 3 + 5 \times 4 \times 3 \times 2$

$= 180$

c) no repeats & only odd no

$4 \times 3 \times \underset{\uparrow}{3} + 4 \times 3 \times 2 \times \underset{1,3,5}{3}$

1, 3, 5

$= 108$

2d) u, v, w, x, y, z

3 or 5 letters

repeat ok

$6 \times 6 \times 6 + 6 \times 6 \times 6 \times 6 \times 6$

$= 7992$

b) no repeats

$6 \times 5 \times 4 + 6 \times 5 \times 4 \times 3 \times 2$

$= 840$

c) no repeats, no 7 first

$5 \times 5 \times 4 + 5 \times 5 \times 4 \times 3 \times 2$

$= 700$

3. 1 → 7

a) no repeat & even

$6 \times 5 \times 4 \times 3 \times 2 \times 1 \times \underset{2,4,6}{3}$

$= 2160$

b) greater than 6000000

∴ & even

$\underset{\text{"6"}}{1} \times 5 \times 4 \times 3 \times 2 \times 1 \times \underset{2,4}{2}$

+

$\underset{\text{"7"}}{1} \times 5 \times 4 \times 3 \times 2 \times 1 \times \underset{2,4,6}{3}$

$= 600$

4. 1 → 5 5 digit code

$= 5! = 120$

a) start 3

$\underset{\text{"3"}}{1} \times 4 \times 3 \times 2 \times 1$

$= 24$

b. end in 5

$4 \times 3 \times 2 \times 1 \quad \underset{\text{"5"}}{1}$

$= 24$

c) start 3 and end 5

$\underset{\text{"3"}}{1} \quad 3 \times 2 \times 1 \quad \underset{\text{"5"}}{1}$

$= 6$

d) start 3 or end 5

$|\text{start 3}| + |\text{end 5}| - |\text{start 3, end 5}|$

$= 24 + 24 - 6$

$= 42$

5. FORECAST no repeats

a) no restrictions

$8! = 40320$

b) E & O together

(EO) FRCAST EO / OE

$= 7! \times 2!$

$= 10080$

c) E & O seperate

$8! - 7!2!$

$= 30240$

d) start A & have EO together

$\underset{\text{"A"}}{1} \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times \underset{\text{EO or OE}}{2!}$

$= 1440$

e) start A or end S

$\underset{\text{"A"}}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times \underset{\text{"S"}}{1} = 5040$

$\underset{\text{"A"}}{1} \; \underset{\text{"S"}}{1} \; 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$|\text{start A}| + |\text{end S}| - |\text{start A, end S}|$

$5040 + 5040 - 720$

$= 9360$

6. 1 → 7 3 digit code no repeats

a) $7 \times 6 \times 5 = 210$

b) start with 4

$\underset{\text{"4"}}{1} \times 6 \times 5 = 30$

c) end in 5

$6 \times 5 \times \underset{\text{"5"}}{1} = 30$

d) start 4 and end 5

$\underset{\text{"4"}}{1} \times 5 \times \underset{\text{"5"}}{1} = 5$

e) start 4 or end 5

$|\text{start 4}| + |\text{end 5}| - |\text{start 4, end 5}|$

$30 + 30 - 5 = 55$

f) odd n°

$6 \times 5 \times \underset{1,3,5,7}{4}$

$= 120$

g) greater than 700

$\frac{1}{\text{"7"}} \times \frac{6}{} \times \frac{5}{} = 30$

h) greater than 500

$\frac{3}{\text{"5,6,7"}} \times \frac{6}{} \times \frac{5}{} = 90$

i) even & greater than 500

$\frac{1}{\text{"5"}} \times \frac{5}{} \times \frac{3}{\text{"2,4,6"}} = 15$

$\frac{1}{\text{"6"}} \times \frac{5}{} \times \frac{2}{\text{2,4}} = 10$

$\frac{1}{\text{"7"}} \times \frac{5}{} \times \frac{3}{\text{2,4,6}} = 15$

40

7. Four people
Terri, Jen, Diane, May

a) Terri at left end

$\frac{1}{\text{Terri}} \times \frac{3}{} \times \frac{2}{} \times \frac{1}{} = 6$

b) Diane at right end

$3 \times \frac{2}{} \times \frac{1}{} \times \frac{1}{\text{Diane}} = 6$

c) Terri left, Diane right

$\frac{1}{\text{Terri}} \times \frac{2}{} \times \frac{1}{} \times \frac{1}{\text{Diane}} = 2$

d) Terri on left or Diane is at right

$|\text{Terri on left}| + |\text{Diane right}| - |\text{Terri left \& Diane right}|$

$6 + 6 - 2 = 10$

e) Jen & Diane in middle two

$\frac{2}{} \times \frac{1}{\text{Jen}} \frac{1}{\text{Di}} \times \frac{1}{}$

$+ \frac{2}{} \times \frac{1}{\text{Diane}} \frac{1}{\text{Jen}} \times \frac{1}{}$

$= 4$

f) Jen & Diane not togetha
= total - when togetha

togetha [Jen Diane] — —
$= 3! \times 2!$

$4! - 3!2!$
$= 12$

8. a) 0 → 8 7 digit odd No
no repeats cannot start 0

$\underset{\text{not 0}}{7} \times 7 \times 6 \times 5 \times 4 \times 3 \times \underset{1,3,5,7}{4}$

pick first

leaves 8 No but cannot use 0

$= 70560$

b) less than 4000 000

$\underset{\text{"4"}}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times \underset{1,3,5,7}{4}$

$\underset{\text{"3"}}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times \underset{1,5,7}{3}$

$\underset{\text{"2"}}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times \underset{1,3,5,7}{4}$

$\underset{\text{"1"}}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times \underset{3,5,7}{3}$

$= 25200$

9. S T T T T T T S
T T D P D T T

$= 2 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 1 \times 4 \times 3 \times 2 \times 1 \times 1 \times 2 \times 1$
$= 14515200$

S T T T T T T S
T T T T
D P D

$2 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 1 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 \times 1$
$= 14515200$

total $= 29030400$

10. a) vowel letters $= 5 \times 26 = 130$

b) start E $\underset{E}{1} \times 26 = 26$

c) end in D $\underset{\text{"D"}}{5} \underset{\text{"D"}}{1} = 5$

d) start E end D $= 1 \times 1 = 1$

e) start E or end D
$|\text{start E}| + |\text{end D}| - |\text{start D end E}|$
$1 \times 26 + 5 \times 1 - 1 \times 1$
$= 26 + 5 - 1 = 30$

f) start end same letter
$5 \quad 1 = 5$
same

g) two different letter
$5 \times 25 = 125$

Ex 25

1. A combintation lock implies you just need to 4 N°
→ they have not implied the order → which is important
∴ rename it to "Permutation lock"

2. {a, b, c, d, e}
$^5C_3 = \frac{5!}{3!2!}$
$= 10$

3. 4 people from 20
$^{20}C_4 = \frac{20!}{16!4!}$
$= 4845$

4. 7 males in total
10 females in total
need 3 males 3 female
$^7C_3 \times {}^{10}C_3$
$= 4200$

5. 2 goalkeepers 5 defenders
↓ ↓
1 3
7 midfield 3 strikers
↓ ↓
4 3
$^2C_1 \times {}^5C_3 \times {}^7C_4 \times {}^3C_3$
$= 700$

6. 6 units to choose in total.
list 1 × 2
list 2 × 1
list 3 × 1
list 4 × 2
$^2C_2 \times {}^3C_1 \times {}^4C_1 \times {}^3C_2$
$= 36$

7. 4 people chosen from 12
$^{12}C_4 = 495$
Chairperson or vicechair but not both
$^1C_1 \, {}^1C_0 \, {}^{10}C_3 + {}^1C_0 \, {}^1C_1 \, {}^{10}C_3$
chair vice chair vice
$= 120 + 120$
$= 240$

8. {a, b, c, d, e, f, g}
Subsets ⇒ empty set → 7 items
$^7C_0 + {}^7C_1 + {}^7C_2 + {}^7C_3 + \ldots {}^7C_7$
$= 1 + 7 + 21 + 35 + 35 + 21 + 7 + 1$
$= 128.$

9. $\{1, 2, \ldots 8, 9\}$ or whole set

all subsets but not empty set

$^9C_1 + {}^9C_2 + {}^9C_3 + \ldots {}^9C_8$

$= 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9$

$= 510$

10. 1 × manager (3) 3 × marketing (15)
1 × engineer (12) 2 × legal. (5)

$^3C_1 \times {}^{12}C_1 \times {}^{15}C_3 \times {}^5C_2$

$= 163800$

a) Joe = engineer Sue = legal

has joe

$^3C_1 \times {}^1C_1 \times {}^{15}C_3 \times {}^5C_2$

$= 13650$

b) Sue

$^3C_1 \times {}^{12}C_1 \times {}^{15}C_3 \times {}^1C_1 \times {}^4C_1$

(Sue)

$= 65520$

c) at least one of Joe or Sue

$|Joe| + |Sue| - |Joe \& Sue|$

$13650 + 65520 - {}^3C_1 \, {}^1C_1 \, {}^{15}C_3 \, {}^1C_1 \, {}^4C_1$

$13650 + 65520 - 5460$

$= 73710$

11. 10 people 4 in committee

a) no restriction

$^{10}C_4 = 210$

b) Javed & Ennis chosen

$^1C_1 \times {}^1C_1 \times {}^8C_2$

(Javed) (Ennis)

$= 28$

c) Connie & Fred or neither of them

$^1C_1 \times {}^1C_1 \times {}^8C_2 + {}^8C_4$

(connie) (Fred) (neither)

$28 + 70$

$= 98$

d) Betty or Henry but not both

$^1C_1 \, {}^1C_0 \, {}^8C_3 + {}^1C_0 \, {}^1C_1 \, {}^8C_3$

(betty) (Henry) (Betty)

$+ {}^1C_0 \times {}^1C_0 \, {}^8C_4$

(Betty) (Henry)

$= 56 + 56 + 70$

$= 182$

12. 8 women 6 men
need committe of 7

a) 4 women, 3 men
$^8C_4 \times {}^6C_3 = 1400$

b) all women.
$^8C_7 = 8$

c) all men → not possible
as there are only 6 men
$= 0$

d) more than 5 women
ie 6 or 7 women
6 women & 1 man $^8C_6\ {}^6C_1$
7 woman & 0 man $^8C_7\ {}^6C_0$
Sum these

$168 + 8$
$= 176$

e) more men than women
6 men 1 woman $^6C_6\ {}^8C_1$
5 men 2 woman $^6C_5\ {}^8C_2$
4 men 3 woman $^6C_4\ {}^8C_3$
sum these

$8 + 168 + 840$

$= 1016$

13 normal deck of cards
$= 52$ cards

a) 8 cards
$^{52}C_8 =$

752538150

b) has Jack of ♡
$^1C_1\ {}^{51}C_7$
↑
J♡
$= 115775100$

c) 5 red & 3 black
$^{26}C_5 \times {}^{26}C_3$

$= 171028000$

d) exactly 2 Queen
$^4C_2\ {}^{48}C_6$
↑
remove Queens

$= 73629072$

e) at least 2 Queen
2Q, 3Q or 4Q
$^4C_2\ {}^{48}C_6 + {}^4C_3\ {}^{48}C_5$
$+ {}^4C_4\ {}^{48}C_4$
$= 8067\,2868$

14. 4 places, 13 to choose from.

6 → Division A 4 → Div B 3 → Div C

a) Choose any 4 $^{13}C_4 = 715$

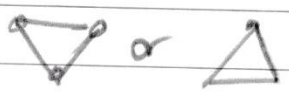
b) at least 1 from each group

A	B	C
2	1	1
1	2	1
1	1	2

$^6C_2\,^4C_1\,^3C_1 + {}^6C_1\,^4C_2\,^3C_1 + {}^6C_1\,^4C_1\,^3C_2$

$= 180 + 108 + 72$

$= 360$

15.  or

top row → 5 bottom row → 4

a) $^5C_2\,^4C_1 + {}^5C_1\,^4C_2$

$= 40 + 30$

$= 70$

b) must include A 

$^1C_1\,^4C_1\,^4C_1 + {}^1C_1\,^4C_2$

$= 16 + 6 = 22$

16. CANDLEPOWR
E

10 actual letters

3 letter word

no E $^9C_4 \times 4! = 3024$

1 E → $1 \times {}^9C_3 \times 4! = 2016$

2 E → $1 \times 1 \times {}^9C_2 \times \frac{4!}{2!}$ ← double E

$= 432$

$3024 + 2016 + 432$

$= 5472.$

16. Equilateral

equilatr
e La

different 8 letters $^8C_3 = 56$

$\times 3! = 336$

2 "E" $^2C_2\,^7C_1 \times \frac{3!}{2!} = 21$

↑ repeated e's

2 "a" $^2C_2\,^7C_1 \times \frac{3!}{2!} = 21$

2 "i" $^2C_2\,^7C_1 \times \frac{3!}{2!} = 21$

$336 + 21 + 21 + 21$

$= 399$

Ex 2F.

1. ${}^nC_r = {}^nC_{n-r}$

LHS:

$${}^nC_r = \frac{n!}{(n-r)! \times r!}$$

RHS:

$${}^nC_{n-r} = \frac{n!}{(n-(n-r))!\,(n-r)!}$$

$$= \frac{n!}{r!\,(n-r)!}$$

LHS = RHS

2.

$${}^1C_0 \quad {}^1C_1$$

$${}^2C_0 \quad {}^2C_1 \quad {}^2C_2$$

$$\vdots$$

$${}^nC_r \quad {}^nC_{r-1} \quad \cdots \quad {}^nC_n$$

eg ${}^4C_1$

${}^5C_1 \quad {}^5C_2$

$\Rightarrow$ ${}^nC_r$ and ${}^nC_{r+1}$ → ${}^{n+1}C_{r+1}$

${}^{n+1}C_r$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

$${}^{n}C_{r+1} = \frac{n!}{(n-(r+1))!\,(r+1)!}$$

$${}^{n+1}C_{r+1} = \frac{(n+1)!}{((n+1)-(r+1))!\,(r+1)!}$$

$$= \frac{(n+1)!}{(n-r)!\,(r+1)!}$$

prove ${}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}$

LHS: $\frac{n!}{(n-(r-1))!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$

$= \frac{n!}{(n-r+1)!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$

$= \frac{n(n-1)(n-2)\ldots}{(n-r+1)(n-r)(n-r-1)\ldots(r-1)(r-2)\ldots}$

$+ \frac{n(n-1)(n-2)\ldots}{(n-r)(n-r-1)(n-r-2)\ldots r(r-1)\ldots}$

⇒ need common denominator of

$(n-r+1)(n-r)(n-r-1)\ldots \times r(r-1)(r-2)\ldots$

⇒ $(n-r+1)!\,r!$

numerator:

$(r)(n)(n-1)\ldots + (n-r+1)(n)(n-1)\ldots$

$(r)(n!) + (n-r+1)(n!)$

factor out $n!$

$n!\,(r + (n-r+1))$

$= n!\,(n+1)$

$= (n+1)!$

$\therefore \frac{(n+1)!}{(n-r+1)!\,r!}$

RHS ${}^{n+1}C_{r}$

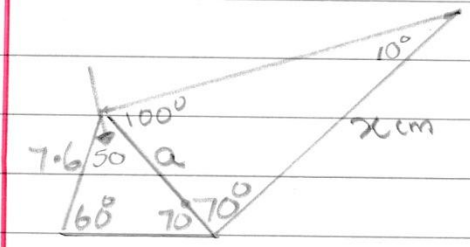
$= \frac{(n+1)!}{(n+1-r)!\,r!}$

$\Rightarrow \frac{(n+1)!}{(n-r+1)!\,r!}$

LHS $\Leftrightarrow$ RHS

Misc Ex 2.

1.



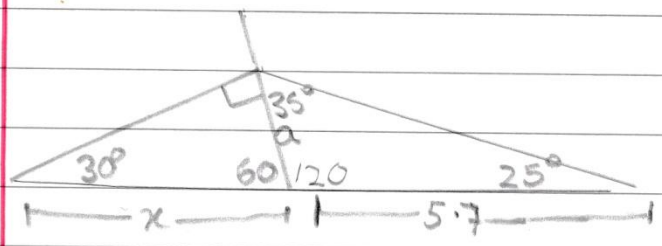
$$\frac{a}{\sin 60} = \frac{7.6}{\sin 70}$$

$$a = 7.00\text{cm}$$

$$\frac{x}{\sin 100} = \frac{7.00}{\sin 10}$$

$$x = 39.7\text{cm}$$

2.



$$\frac{a}{\sin 25} = \frac{5.7}{\sin 35}$$

$$a = 4.1998$$

$$\cos 60 = \frac{4.1998}{x}$$

$$x = 8.4\text{cm}$$

3. $x = 8$ then $x^2 = 64$

converse:

if $x^2 = 64$ then $x = 8$

false as x is also -8

contrapositive

if $x^2 \neq 64$ then $x \neq 8$

true.

4. FISH.

there will be

$4 \times 3 \times 2 \times 1 = 24$

options possible

25 in class.

$n+1 > n$

∴ by P.H.P

there must be

at least **2** student

with the same answer

5.

race 8	race 9
1st 2nd 3rd	1st 2nd
8 horses	12 horses

make 5 places

$8 \times 7 \times 6 \times 12 \times 11$

$= 44352$

6. a, b, c, d, e, f, g, h, i.

9 letters ⇒ make 5 letter word

a) $9 \times 8 \times 7 \times 6 \times 5 = 15120$

b) 2 vowels 3 consonants

$^3C_2 \times {}^6C_3 \times 5!$

$3 \times 20 \times 120$

$= 7200$

7. Bring 5 toys from
6 jigsaw
8 doll
4 ball
2 truck.
} 20 toys

a) $^{20}C_5 = 15504$

b) must have at least 1 from each category

$^6C_2 \times {}^8C_1 \times {}^4C_1 \times {}^2C_1 = 960$

(jig saw, doll, ball, truck)

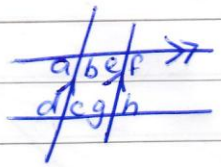
$^6C_1 \times {}^8C_2 \times {}^4C_1 \times {}^2C_1 = 1344$

$^6C_1 \times {}^8C_1 \times {}^4C_2 \times {}^2C_1 = 576$

$^6C_1 \times {}^8C_1 \times {}^4C_1 \times {}^2C_2 = 192$

$= 960 + 1344 + 576 + 192 = 3072$

8.



$a = c$ $e = h$ } alternate
$b = d$ $f = g$ } angles in parallel lines

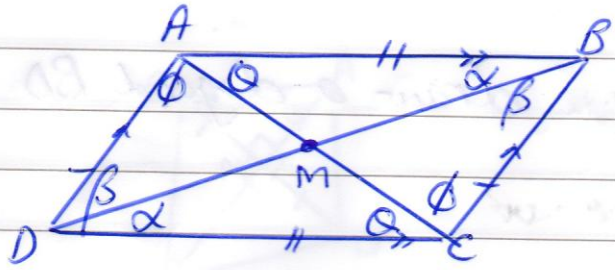
$a + b = 180$ $e + f = 180$
$c + d = 180$ $g + h = 180$
angles in a straight line are supplementary

$b + c = 180$ $b + e = 180$
$e + g = 180$ $c + g = 180$
co-interior angles are supplementary

∴ from algebraic deduction (not shown here)
$b = g$ & $e = c$

∴ quadrilateral formed is a parallelogram whose opposite pairs of sides are equal in length.

9.



Shape is a parallelogram. (given in the question).

as $AB \parallel DC$ (given)
$\angle BAC = \angle ACD$ (alternate Z angles) (θ)

as $AD \parallel BC$
$\angle ABD = \angle BDC$ (alternate Z angles) (α)

$\therefore \triangle AMB \equiv \triangle CMD$ (ASA)

$\therefore AM = MC$ ie AC is bisected at point M

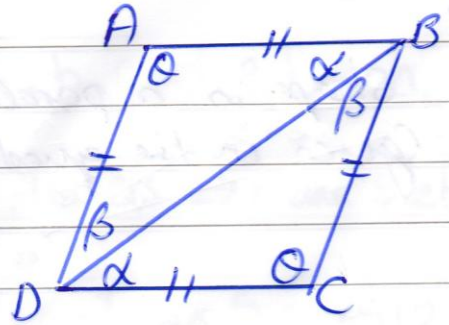
as $AD \parallel BC$
$\angle DAC = \angle BCA$ (alternate Z angles) (ϕ)

as $AD \parallel BC$
$\angle ADB = \angle DBC$ (alternate Z angles) (β)

$\therefore \triangle AMD \equiv \triangle CMB$ (A.S.A)
$\therefore DM = BM$ ie M bisects DM.

$\therefore$ diagonals of a parallelogram bisect each other

0.



draw a diagonal BD

In $\triangle ABD$ & In $\triangle BCD$

AB	=	DC	(given)
AD	=	CB	(given)
DB	=	DB	(common side)

$\therefore \triangle BAD \equiv \triangle BCD$ (S.S.S)

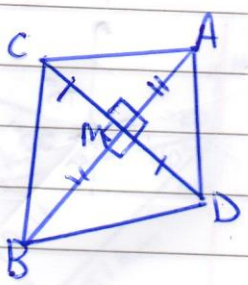
$\therefore \angle BAC = \angle BCD$ (θ)
$\angle ABD = \angle BDC$ (α)
$\angle ADB = \angle DBC$ (β)

$\therefore$ For $\alpha = \alpha$ & $\beta = \beta$ they must belong on lines AB & DC which need to be parallel in order for them to be alternate angles

$\therefore$ AB // DC & AD // BC

$\therefore$ ABCD is also a parallelogram.

11.



AB ⊥ CD given

AB & CD bisect ie AM = MB (given)
ie CM = MD

In $\triangle AMD$ & In $\triangle DMB$

$AM = BM$ (AB is bisected)
$MD = MD$ (common side)
$\angle AMD = \angle DMB$ (90° given)

$\therefore \triangle AMD \equiv \triangle BMD$ (S.A.S)
$\therefore AD = BD$

In $\triangle AMC$ & $\triangle BMC$

$AM = BM$ (AB is bisected)
$CM = CM$ (common side)
$\angle AMC = \angle BMC$ (90° given)
$\therefore \triangle AMC \equiv \triangle BMC$ (S.A.S
$\therefore AC = BC$

In $\triangle DMB$ & $\triangle BMC$

$Bm = BM$ (common side)
$DM = CM$ (CD is bisected
$\angle DMB = \angle BMC$ (90° given)
$\therefore \triangle DMB \equiv \triangle CMB$ (S.A.S)
$\therefore BD = CB$
$\therefore AD = DB = BC = CA$.